



Properties of Logarithms

Allow us to write log expressions in various forms

$$\log(xy) = \log x + \log y \quad (\text{Power Rule})$$

$$\log\left(\frac{x}{y}\right) = \log x - \log y \quad (\text{Quotient Rule})$$

$$\log x^n = n \log x \quad (\text{Power Rule})$$

* Use the log properties to expand and condense expressions

Expand example

$$\begin{aligned} \log x^2 y &= \log x^2 + \log y \\ &= 2 \log x + \log y \end{aligned}$$

Condense example

$$\begin{aligned} 2 \log x + \log y &= \log x^2 + \log y \\ &= \log x^2 y \end{aligned}$$



The Natural Base e

Any power where the base is the number "e"

$$e \approx 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \dots$$

$$e \approx 2.718281828\dots$$

* many things in nature can be modeled with exponential functions of base e.



Natural Logarithms

logarithms that are base e

$$\log_e x = \ln x$$



we write as



* all log properties apply

Example:

$$\begin{aligned} \text{Expand } \ln y^3 z &= \ln y^3 + \ln z \\ &= 3 \ln y + \ln z \end{aligned}$$



Solving Logarithmic Equations

* Key idea - use exponents to solve

Steps

1. Condense log expressions (as necessary) to get the equation in the form

$$\log_a x = y$$

examples $\log x = 9$

$$\ln x = 12$$

$$\log_b \frac{6}{(x+1)} = 2$$

2. Rewrite the log equation as an exponential equation

examples $\log x = 9 \longrightarrow 10^9 = x$

$$\ln x = 12 \longrightarrow e^{12} = x$$

$$\log_b \frac{6}{(x+1)} = 2 \longrightarrow \underbrace{(x+1)^2 = 6}$$

solve these equations



Solving Exponential Equations

* Key idea - use logs to solve

Steps

1. Take necessary steps to isolate the exponential part of the equation

$$a^x = b$$

examples

$$3^x - 2 = 6$$
$$3^x = 8$$
$$2e^x = 10$$
$$e^x = 5$$

2. take the log of both sides

use properties of logs

$$\left\{ \begin{array}{l} \log 3^x = \log 8 \\ x \log 3 = \log 8 \\ x = \frac{\log 8}{\log 3} \end{array} \right\} \text{ use calculator}$$

when base e
use ln

$$\ln e^x = \ln 5$$
$$x \ln e = \ln 5$$
$$x = \ln 5 \quad \text{use calculator}$$