Radical Expressions and Equations

Chapter Review

Simplifying Radicals

- Look for perfect square factors

Perfect squares- 4, 9, 16, 25, 36, ...
$\sqrt{4}$ $\sqrt{9}$ $\sqrt{16}$ $\sqrt{25}$ $\sqrt{36}$
2 3 4 5 6 ....

Example

$$\sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$$

↑ Factor ↑ seperate radicals

Operations with Radicals

- Adding and subtracting - very similar to adding and subtracting like terms

Examples

$3\sqrt{7} + 2\sqrt{7} = 5\sqrt{7}$

↑ SAME ↑

Note- always simplify radical terms before adding and subtracting

$3\sqrt{7} + 2\sqrt{6}$

↖ ↗

Not the same - can't add

• Multiplying / Dividing radicals - follow these rules

$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ Example $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ Example $\sqrt{\frac{16}{23}} = \frac{\sqrt{16}}{\sqrt{23}} = \frac{4}{\sqrt{23}}$

Solving Radical Equations

Steps

1. isolate radical
2. raise power to both sides - clear radical
3. Solve and check solution - disregard extraneous solutions (these solutions produce false statements when you verify)

Example $2\sqrt{x+1} - 3 = 15$

Step 1

$$2\sqrt{x+1} - 3 = 15$$
$$+3 \quad +3$$
$$\frac{2\sqrt{x+1}}{2} = \frac{18}{2}$$
$$\sqrt{x+1} = 9$$

Step 2

$$(\sqrt{x+1})^2 = 9^2$$
$$x + 1 = 81$$
$$x = 80$$

Step 3

$$2\sqrt{x+1} - 3 = 15$$

↑ plug in $x = 80$

$$2\sqrt{80+1} - 3 = 15$$
$$2\sqrt{81} - 3 = 15$$
$$2 \cdot 9 - 3 = 15$$
$$18 - 3 = 15$$
$$15 = 15$$

True ↗

Therefore $x = 80$ is the solution

Distance and Mid-Point Formula

- Distance formula finds the distance between two points
- Mid-point formula finds the point that is halfway between two points

Distance Formula

$$d = \sqrt{(x - x_1)^2 + (y - y_1)^2}$$

Example, find the distance between (2,4) and (5,10)

$$d = \sqrt{(x - x_1)^2 + (y - y_1)^2}$$

$x = 2$, $x_1 = 5$, $y = 4$, $y_1 = 10$

$$d = \sqrt{(2-5)^2 + (4-10)^2}$$

$$= \sqrt{(-3)^2 + (-6)^2}$$

$$= \sqrt{9 + 36}$$

$$d = \sqrt{45} \approx 6.70$$

Mid-point formula $\left(\frac{x+x_1}{2}, \frac{y+y_1}{2}\right)$

Example. find the mid-point between $(6,2)$ and $(2,10)$

$$\left(\frac{x+x_1}{2}, \frac{y+y_1}{2}\right) = \left(\frac{6+2}{2}, \frac{2+10}{2}\right)$$

$$= \left(\frac{8}{2}, \frac{12}{2}\right)$$

$(4,6)$ is halfway between $(6,2)$, $(2,10)$

$MP = (4,6)$

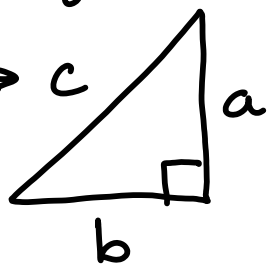
The Pythagorean Theorem

- Finds the length of the legs of a right triangle
- If you know two of the lengths of a right triangle you can find the third

The Pythagorean Theorm

Given right triangle

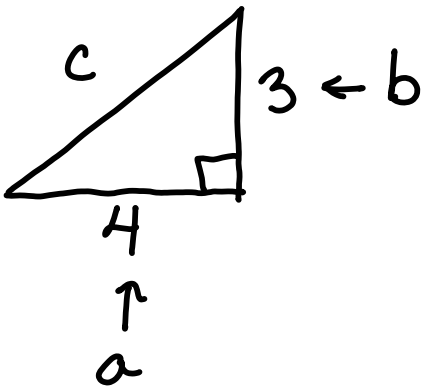
Note - c is always the longest leg



$$a^2 + b^2 = c^2$$

Examples,

Find the length of the missing legs



$a^2 + b^2 = c^2$

$4^2 + 3^2 = c^2$

$16 + 9 = c^2$

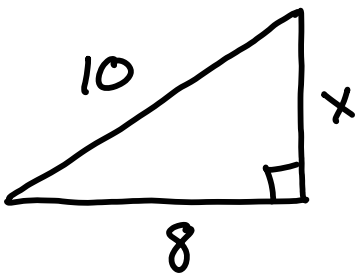
$25 = c^2$

Find c by taking square root of both sides

$c^2 = 25$

$\sqrt{c^2} = \sqrt{25}$

$c = 5$ ← answer



$c = 10$
$a = 8$
$b = x$

plug-in

$a^2 + b^2 = c^2$

$8^2 + x^2 = 10^2$

$64 + x^2 = 100$

$-64 \quad -64$ ← get x^2 isolated

$x^2 = 36$

$\sqrt{x^2} = \sqrt{36}$

$x = 6$ ← answer