Rational Expressions and Equations

Chapter Review

Ratios and Proportion

<u>Ratio</u> – comparing two numbers by division, the numbers count the same concept / usually same unit of measure

Example $\dfrac{1 \text{ Teacher}}{20 \text{ students}}$

<u>Rate</u> – comparing two numbers by division, the numbers count different concepts / usually different unit of measure

<u>Proportion</u> – two equal ratios or rates

- The cross-products are equal

Example $\dfrac{1}{2} = \dfrac{3}{6}$ cross-product

$$1 \cdot 6 = 2 \cdot 3$$
$$6 = 6$$

- Use the cross-product to solve various rate and ratio problems

Percent

- Percent – is a ratio that compares a number to 100.

Example, $\frac{70}{100}$ is 70 percent = 70%

Find percent of a number

- Change percent to decimal, multiply by number

Example 60% of 90

↓

Divide by 100 to change to decimal

$(.60)(90) = 54$

Other percent problems

- Use an equation or proportion to solve

Example, 15 is what percent of 75?

Equation method –

this is the percent – in decimal form we are looking for

$X \cdot 75 = 15$

$75x = 15$

$x = .2$

Change decimal to % multiply by 100

Answer

$.2 \times 100 = 20\%$

Percent - proportion method

looking for % ⟶ $\frac{x}{100} = \frac{15}{75}$

- Use cross-product to solve

$75x = 100(15)$

$75x = 1500$

$x = 20\%$

Direct and Inverse Variation

$y = kx$ Direct $\qquad$ $yx = k$ Inverse

K = constant of variation

Steps to solve variation problems

1. Identify if the relationship between the variables is direct or inverse
2. Use the general variation formula and information in problem to solve for k
3. Write a specific equation with the k-value
4. Use the equation from step 3 to solve the problem

Example, x and y vary directly, when $x = 3$ $y = 6$. What is y when $x = 10$?

Step 1. $y = kx$

Step 2. $6 = K3$

Solve for $k = 2$

Step 3. $y = 2x$

Step 4. $y = 2(10)$

$y = 20$ Answer

Simplifying Rational Expressions

- Cancel out like factors
- Know how to factor

like factors

$$\frac{10x^2y}{15x^5y^2} = \frac{2\cdot\cancel{5}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{y}}{3\cdot\cancel{5}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot x\cdot\cancel{y}\cdot y} = \frac{2}{3x^3y}$$

Factor

$$\frac{2x+8}{(x-4)(x+4)}$$

$$= \frac{2\cancel{(x+4)}}{(x-4)\cancel{(x+4)}} = \frac{2}{x-4}$$

like factors

Multiplying and Dividing Rational Expressions

- Follow same rules as numeric fractions
- Know how to multiply polynomials
- Always simplify your results

Examples,

$$\frac{3x}{x+1}\cdot\frac{(2x+4)}{5} = \frac{3x(2x+4)}{5(x+1)} = \frac{6x^2+12x}{5x+5}$$

$$\frac{3x}{x+1}\div\frac{(2x+4)}{5} = \frac{3x}{(x+1)}\cdot\frac{5}{(2x+4)} =$$

Flip, make problem into multiplication

$$\frac{15x}{(x+1)(2x+4)}$$

Adding and Subtracting Rational Expressions

- Follow the same rules as adding / subtracting fractions
- Know how to find the LCD of a rational expression
- Simplify your results

Examples,

$$\frac{7x}{5x-3} - \frac{x}{5x-3} = \frac{7x-x}{5x-3} = \frac{6x}{5x-3}$$

same denominators, subtract numerators

$$\frac{2x}{(x-1)(x+3)} + \frac{5}{(x+3)}$$

Not common denominators, to fix multiply $(x-1)$ to the numerator / denominator of

$$\frac{5}{(x+3)} \cdot \frac{(x-1)}{(x-1)} = \frac{5x-5}{(x+3)(x-1)}$$

$$\frac{2x}{(x-1)(x+3)} + \frac{5x-5}{(x+3)(x-1)}$$

Now we can add

$$= \frac{2x+5x-5}{(x-1)(x+3)} = \frac{7x-5}{(x-1)(x+3)}$$

Answer

Solving Rational Equations

Steps

1. Clear fractions by multiplying both sides of the equation by the LCD

2. Solve the remaining equations
3. Check your solutions - ignore "extraneous" solutions (any solution that creates a false statement when you check it)

* When you have one single-fraction rational expression equal to another you may use the cross-product to solve

Examples $\dfrac{x+1}{4} = \dfrac{3}{6}$

use cross-product $6(x+1) = 12$

$6x + 6 = 12$

$6x = 6$

$x = 1$

check

$x=1 \rightarrow \dfrac{x+1}{4} = \dfrac{3}{6}$

$\dfrac{2}{4} = \dfrac{3}{6} = \dfrac{1}{2}$ True, $x=1$ is the solution

$$\frac{x}{x+2} + \frac{1}{3} = 4$$

Clear fractions - LCD $= 3(x+2)$

$$3(x+2)\left(\frac{x}{x+2}+\frac{1}{3}=4\right)$$

$$3\cancel{(x+2)}\cdot\frac{x}{\cancel{(x+2)}}+\cancel{3}(x+2)\cdot\frac{1}{\cancel{3}}=3(x+2)\cdot 4$$

$$3x + x+2 = 12(x+2)$$

$$4x+2=12x+24$$

$$-8x=22$$

$$x=-\frac{22}{8}=-\frac{11}{4}$$

Check $x=-11/4$ in original equation

$$-11/4 \rightarrow \frac{x}{x+2}+\frac{1}{3}=4$$

$$\frac{-11/4}{(-11/4+2)}+\frac{1}{3}=4$$

$$\frac{\frac{-11}{4}}{-\frac{3}{4}}+\frac{1}{3}=4$$

CAN not skip verifying the solution(s); sometimes you will get false solutions - must check!

$$\frac{-11}{4}\div-\frac{3}{4}=\rightarrow\frac{11}{3}+\frac{1}{3}=4$$

$$\frac{12}{3}=4=4$$

True, left = right

$x=-11/4$ is the solution